Question 1: If α and β are the zeros of the quadratic polynomial $f(x) = x^2 - 5x + 4$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$.

ANSWER:

Since α and β are the zeros of the quadratic polynomial $f(x) = x^2 - 5x + 4$

Therefore $\alpha + \beta = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

$= \frac{-(5)}{1}$

= 5

$\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

$$= \frac{4}{1}$$
$$= 4$$

We have, $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$

$$\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta = \frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$$
$$\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta = \frac{\alpha+\beta}{\alpha\beta} - 2\alpha\beta$$

By substituting $\alpha + \beta = 5$ and $\alpha\beta = 4$ we get ,

$$\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta = \frac{5}{4} - 2(4)$$
$$\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta = \frac{5}{4} - \frac{8\times 4}{1\times 4}$$

Taking least common factor we get ,

$$\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta = \frac{5-32}{4}$$
$$\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta = \frac{-27}{4}$$

Hence, the value of $\frac{1}{\alpha}+\frac{1}{\beta}-2\alpha\beta$ is $\boxed{\frac{-27}{4}}$.

Question 2: If α and β are the zeros of the quadratic polynomial $f(x) = x^2 - x - 4$, find the value of $\frac{1}{\alpha}+\frac{1}{\beta}-\alpha\beta$.

ANSWER:

Since α and β are the zeros of the quadratic polynomials $f(x)=x^2-x-4$

sum of the zeros = $\frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

$$\alpha+\beta=-\left[-\frac{1}{1}\right]$$
$$\alpha+\beta=\frac{1}{1}$$
$$\alpha+\beta=1$$

Product if zeros = $\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

$$\alpha\beta=\frac{-4}{1}$$
$$\alpha\beta=-4$$

We have,

$$\frac{1}{\alpha}+\frac{1}{\beta}-\alpha\beta$$
$$\frac{\alpha+\beta}{\alpha\beta}-\alpha\beta$$

By substituting $\alpha+\beta=1$ and $\alpha\beta=-4$ we get ,

$$\frac{1}{\alpha}+\frac{1}{\beta}-\alpha\beta=\frac{1}{-4}-(-4)$$
$$\frac{1}{\alpha}+\frac{1}{\beta}-\alpha\beta=\frac{1}{-4}+\frac{4}{1}$$
$$\frac{1}{\alpha}+\frac{1}{\beta}-\alpha\beta=\frac{1}{-4}+\frac{4\times4}{1\times4}$$
$$\frac{1}{\alpha}+\frac{1}{\beta}-\alpha\beta=\frac{1}{-4}+\frac{16}{4}$$

$$\frac{1}{\alpha}+\frac{1}{\beta}-\alpha\beta=\frac{-1+16}{4}$$
$$\frac{1}{\alpha}+\frac{1}{\beta}-\alpha\beta=\frac{15}{4}$$

Hence, the value of $\frac{1}{\alpha}+\frac{1}{\beta}-\alpha\beta$ is $\boxed{\frac{15}{4}}$.

Question 3: If α and β are the zeros of the quadratic polynomial *f*(*x*) = x^2 + *x* − 2, find the value of $\frac{1}{\alpha}-\frac{1}{\beta}$.

ANSWER:

Since α and β are the zeros of the quadratic polynomials $f(x)=x^2+x-2$

Sum of the zeros = $\frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

$$\alpha+\beta=-\left[\frac{1}{1}\right]$$
$$\alpha+\beta=-\frac{1}{1}$$
$$\alpha+\beta=-1$$

Product if zeros = $\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

$$\alpha\beta=\frac{-2}{1}$$

$$\alpha\beta=-2$$

We have, $\frac{1}{\alpha}-\frac{1}{\beta}$

$$\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^2=\left(\frac{1}{\alpha}\right)^2+\left(\frac{1}{\beta}\right)^2+\frac{2}{\alpha\beta}$$
$$\left(\frac{\alpha+\beta}{\alpha\beta}\right)^2=\left(\frac{1}{\alpha}\right)^2+\left(\frac{1}{\beta}\right)^2+\frac{2}{\alpha\beta}$$
$$\left(\frac{\alpha+\beta}{\alpha\beta}\right)^2-\frac{2}{\alpha\beta}=\left(\frac{1}{\alpha}\right)^2+\left(\frac{1}{\beta}\right)^2$$

By substituting $\alpha+\beta=-1$ and $\alpha\beta=-2$ we get ,

$$\left(\frac{-1}{-2}\right)^2-\frac{2}{-2}=\left(\frac{1}{\alpha}\right)^2+\left(\frac{1}{\beta}\right)^2$$

$$\frac{1}{4}+1=\left(\frac{1}{\alpha}\right)^2+\left(\frac{1}{\beta}\right)^2$$

$$\frac{1}{4}+\frac{1\times 4}{1\times 4}=\left(\frac{1}{\alpha}\right)^2+\left(\frac{1}{\beta}\right)^2$$

$$\frac{1+4}{4}=\left(\frac{1}{\alpha}\right)^2+\left(\frac{1}{\beta}\right)^2$$

$$\frac{5}{4}=\left(\frac{1}{\alpha}\right)^2+\left(\frac{1}{\beta}\right)^2$$

$$\left(\frac{1}{\alpha}-\frac{1}{\beta}\right)^2=\left(\frac{1}{\alpha}\right)^2+\left(\frac{1}{\beta}\right)^2-\frac{2}{\alpha\beta}$$

By substituting $\frac{5}{4}=\left(\frac{1}{\alpha}\right)^2+\left(\frac{1}{\beta}\right)^2$ in $\left(\frac{1}{\alpha}\right)^2+\left(\frac{1}{\beta}\right)^2-\frac{2}{\alpha\beta}$ we get ,

$$\left(\frac{1}{\alpha}-\frac{1}{\beta}\right)^2=\frac{5}{4}-\frac{2}{-2}$$

$$\left(\frac{1}{\alpha}-\frac{1}{\beta}\right)^2=\frac{5}{4}+1$$

$$\left(\frac{1}{\alpha}-\frac{1}{\beta}\right)^2=\frac{5}{4}+\frac{1\times 4}{1\times 4}$$

$$\left(\frac{1}{\alpha}-\frac{1}{\beta}\right)^2=\frac{5+4}{4}$$

$$\left(\frac{1}{\alpha}-\frac{1}{\beta}\right)^2=\frac{9}{4}$$

Taking square root on both sides we get

$$\sqrt{\left(\frac{1}{\alpha}-\frac{1}{\beta}\right)^2}=\sqrt{\frac{3\times 3}{2\times 2}}$$
$$\frac{1}{\alpha}-\frac{1}{\beta}=\pm\frac{3}{2}$$
.

Hence, the value of $\frac{1}{\alpha}-\frac{1}{\beta}$ is $\boxed{\pm\frac{3}{2}}$.

Question 4: If the sum of the zeros of the quadratic polynomial *f*(*t*) = *kt*^{2} + 2*t* + 3*k* is equal to their product, find the value of *k*.

ANSWER:

Let α,β be the zeros of the polynomial $f(t)=kt^2+2t+3k$ .Then,

$$\alpha+\beta=\frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$\alpha+\beta=\frac{-2}{k}$$

$$\alpha\beta=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$\alpha\beta=\frac{3k}{k}$$
$$\alpha\beta=\frac{3\cancel{k}}{\cancel{k}}$$
$$\alpha\beta=3$$

It is given that the sum of the zero of the quadratic polynomial is equal to their product then, we have

$$\alpha+\beta=\alpha\beta$$
$$\frac{-2}{k}=3$$
$$-2=3\times k$$
$$\frac{-2}{3}=k$$

Hence, the value of k is $\boxed{\frac{-2}{3}}$

Question 5: If α and β are the zeros of the quadratic polynomial $f(x) = x^2 - 3x - 2$, find a quadratic polynomial whose zeros are $\frac{1}{2\alpha+\beta}$ and $\frac{1}{2\beta+\alpha}$

ANSWER:

Since $\alpha and \beta$ are the zeros of the quadratic polynomial $f(x) = x^2 - 3x - 2$

The roots are α and β

$$\alpha + \beta = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$\alpha + \beta = -\left(\frac{-3}{1}\right)$$
$$\alpha + \beta = -(-3)$$
$$\alpha + \beta = 3$$

$$\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$\alpha\beta = \frac{-2}{1}$$
$$\alpha\beta = -2$$

Let S and P denote respectively the sum and the product of zero of the required polynomial . Then,

$$S = \frac{1}{2\alpha + \beta} + \frac{1}{2\beta + \alpha}$$

Taking least common factor then we have ,

$$S = \frac{1}{2\alpha + \beta} \times \frac{2\beta + \alpha}{2\beta + \alpha} + \frac{1}{2\beta + \alpha} \times \frac{2\alpha + \beta}{2\alpha + \beta}$$
$$S = \frac{2\beta + \alpha}{(2\alpha + \beta)(2\beta + \alpha)} + \frac{2\alpha + \beta}{(2\beta + \alpha)(2\alpha + \beta)}$$
$$S = \frac{2\beta + \alpha + 2\alpha + \beta}{(2\alpha + \beta)(2\beta + \alpha)}$$
$$S = \frac{3\beta + 3\alpha}{4\alpha\beta + 2\beta^2 + 2\alpha^2 + \beta\alpha}$$

$$S=\frac{3(\beta+\alpha)}{5\alpha\beta+2(\alpha^2+\beta^2)}$$

$$S=\frac{3(\beta+\alpha)}{5\alpha\beta+2\left[(\alpha+\beta)^2-2\alpha\beta\right]}$$

By substituting $\alpha+\beta=3$ and $\alpha\beta=-2$ we get ,

$$S=\frac{3(3)}{5(-2)+2\left[(3)^2-2\times-2\right]}$$

$$S=\frac{9}{-10+2(13)}$$

$$S=\frac{9}{-10+26}$$

$$S=\frac{9}{16}$$

$$P=\frac{1}{2\alpha+\beta}\times\frac{1}{2\beta+\alpha}$$

$$P=\frac{1}{(2\alpha+\beta)(2\beta+\alpha)}$$

$$P=\frac{1}{4\alpha\beta+2\beta^2+2\alpha^2+\beta\alpha}$$

$$P=\frac{1}{5\alpha\beta+2(\alpha^2+\beta^2)}$$

$$P=\frac{1}{5\alpha\beta+2\left[(\alpha+\beta)^2-2\alpha\beta\right]}$$

By substituting $\alpha+\beta=3$ and $\alpha\beta=-2$ we get ,

$$P=\frac{1}{5(-2)+2\left[(3)^2-2\times-2\right]}$$

$$P=\frac{1}{10+2[9+4]}$$

$$P=\frac{1}{10+2(13)}$$

$$P=\frac{1}{-10+26}$$

$P = \frac{1}{16}$

Hence ,the required polynomial $f(x)$ is given by

$$f(x) = k(x^2 - Sx + P)$$
$$f(x) = k\left(x^2 - \frac{9}{16}x + \frac{1}{16}\right)$$

Hence, the required equation is $\boxed{f(x) = k\left(x^2 - \frac{9}{16}x + \frac{1}{16}\right)}$ Where *k* is any non zero real number.

Question 6: Find the cubic polynomial with the sum, sum of the product of its zeros taken two at a time, and product of its zeros as 3, −1 and − 3 respectively.

ANSWER:

If α, β and γ are the zeros of a cubic polynomial *f* (*x*), then

$f(x) = k\{x^3 - (\alpha+\beta+\gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma\}$ where *k* is any non-zero real number.

Here,

$$\alpha + \beta + \gamma = 3,$$
$$\alpha\beta + \beta\gamma + \gamma\alpha = -1$$
$$\alpha\beta\gamma = -3$$

Therefore

$$f(x) = k\{x^3 - (3)x^2 + (-1)x - (-3)\}$$
$$f(x) = k\{x^3 - 3x^2 - 1x + 3\}$$

Hence, cubic polynomial is $\boxed{f(x) = k\{x^3 - 3x^2 - 1x + 3\}}$, where *k* is any non-zero real number.